

The role of hyperfine mixing in $b \rightarrow c$ semileptonic and electromagnetic decays of doubly-heavy baryons¹

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Abstract. We analyze the effects of hyperfine mixing in $b \rightarrow c$ semileptonic and electromagnetic decays of doubly heavy baryons.

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INTRODUCTION

In the infinite heavy quark mass limit, and according to heavy quark spin symmetry, the total spin of the heavy quark subsystem in a doubly heavy baryon can be selected to be $S_h = 0, 1$. Indeed, this has been the criterion for the most common classification scheme of these baryons. Table 1 summarizes the quark content and quantum numbers of the baryons considered in this study, classified so that S_h is well defined, and to which we shall refer as the S_h -basis. Being ground state baryons, a total orbital angular momentum $L = 0$ is assumed.

TABLE 1. Quantum numbers and quark content of ground-state doubly heavy baryons.

Baryon	Quark content (l=u,d)	S_h	J^π	Baryon	Quark content	S_h	J^π
Ξ_{cc}	{c c} 1	1	1/2 ⁺	Ω_{cc}	{c c} s	1	1/2 ⁺
Ξ_{cc}^*	{c c} 1	1	3/2 ⁺	Ω_{cc}^*	{c c} s	1	3/2 ⁺
Ξ_{bb}	{b b} 1	1	1/2 ⁺	Ω_{bb}	{b b} s	1	1/2 ⁺
Ξ_{bb}^*	{b b} 1	1	3/2 ⁺	Ω_{bb}^*	{b b} s	1	3/2 ⁺
Ξ_{bc}	{b c} 1	1	1/2 ⁺	Ω_{bc}	{b c} s	1	1/2 ⁺
Ξ_{bc}^*	{b c} 1	1	3/2 ⁺	Ω_{bc}^*	{b c} s	1	3/2 ⁺
Ξ_{bc}^{\prime}	[b c] 1	0	1/2 ⁺	Ω_{bc}^{\prime}	[b c] s	0	1/2 ⁺

Due to the finite value of the heavy quark masses, the hyperfine interaction between

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TABLE 2. Masses (in MeV) for unmixed states.

	This work	[5]	[1]		This work	[5]	[1]
$M_{\Xi_{cc}}$	3613	3620	3676	$M_{\Omega_{cc}}$	3712	3778	3815
$M_{\Xi_{cc}^*}$	3707	3727	3753	$M_{\Omega_{cc}^*}$	3795	3872	3876
$M_{\Xi_{bb}}$	10198	10202	10340	$M_{\Omega_{bb}}$	10269	10359	10454
$M_{\Xi_{bb}^*}$	10237	10237	10367	$M_{\Omega_{bb}^*}$	10307	10389	10486
$M_{\Xi_{bc}}$	6928	6933	7020	$M_{\Omega_{bc}}$	7013	7088	7147
$M_{\Xi'_{bc}}$	6958	6963	7044	$M_{\Omega'_{bc}}$	7038	7116	7166
$M_{\Xi_{bc}^*}$	6996	6980	7078	$M_{\Omega_{bc}^*}$	7075	7130	7191

the light quark and any of the heavy quarks can admix both $S_h = 0$ and $S_h = 1$ spin components into the wave function. This mixing is negligible for cc and bb baryons, as the antisymmetry of the wave function would require higher orbital angular momenta or radial excitations. On the other hand, in the bc sector, one would expect the role of the mixing to be noticeable and actual physical states to be admixtures of the B_{bc} and B'_{bc} ($B = \Xi, \Omega$) states listed in Table 1.

Masses are rather insensitive to the mixing, and most calculations simply ignore it and use the S_h -basis. Roberts and Pervin [1] took the issue of the hyperfine mixing and its role in semileptonic decays. They noticed that, working in the S_h -basis, the decay width of Ξ_{bc} (Ω_{bc}) greatly differs from that of Ξ'_{bc} (Ω'_{bc}) so that mixing could be of relevance for these processes. In Ref. [2] they found that indeed mixing had an enormous impact in semileptonic decays of doubly-heavy baryons.

We have studied the effect of this mixing in both semileptonic and electromagnetic decays in Refs. [3] and [4]. Our study on the semileptonic decay widths of bc baryons qualitatively corroborate the findings of Roberts and Pervin. In the electromagnetic case the decay widths are proportional to $(M_I - M_F)^3$, with M_I, M_F the initial and final baryon masses, showing thus a strong dependence on the actual baryon masses.

RESULTS AND DISCUSSION

Table 2 shows our results for the masses of the unmixed states. We compare them to the results from Refs. [5] and [1]. Details on the model used can be found in Ref. [6]. Mixed bc states are obtained by diagonalizing the corresponding mass matrices. In our calculation, the mixed states and masses are given by [3]

$$\begin{aligned}
\Xi_{bc}^{(1)} &= 0.902 \Xi'_{bc} + 0.431 \Xi_{bc}, \quad M_{\Xi_{bc}^{(1)}} = 6967 \text{ MeV}, \\
\Xi_{bc}^{(2)} &= -0.431 \Xi'_{bc} + 0.902 \Xi_{bc}, \quad M_{\Xi_{bc}^{(2)}} = 6919 \text{ MeV}, \\
\Omega_{bc}^{(1)} &= 0.899 \Omega'_{bc} + 0.437 \Omega_{bc}, \quad M_{\Omega_{bc}^{(1)}} = 7046 \text{ MeV}, \\
\Omega_{bc}^{(2)} &= -0.437 \Omega'_{bc} + 0.899 \Omega_{bc}, \quad M_{\Omega_{bc}^{(2)}} = 7005 \text{ MeV}.
\end{aligned} \tag{1}$$

Comparing with Table 2, we see small changes in the masses when mixing is taken into account. However, as shown in Eq. (1), the admixture is important and it can affect the

TABLE 3. Semileptonic decay widths (in units of 10^{-14} GeV) for unmixed states. We use $|V_{cb}| = 0.0413$. $l = e, \mu$.

	This work	[7]	[8]	[2]		This work	[7]	[8]	[2]
$\Gamma(\Xi_{bb}^* \rightarrow \Xi'_{bc} l \bar{\nu}_l)$	1.08	0.82	0.36	–	$\Gamma(\Omega_{bb}^* \rightarrow \Omega'_{bc} l \bar{\nu}_l)$	1.14	0.85	0.42	–
$\Gamma(\Xi_{bb}^* \rightarrow \Xi_{bc} l \bar{\nu}_l)$	0.36	0.28	0.14	–	$\Gamma(\Omega_{bb}^* \rightarrow \Omega_{bc} l \bar{\nu}_l)$	0.38	0.29	0.15	–
$\Gamma(\Xi_{bb} \rightarrow \Xi'_{bc} l \bar{\nu}_l)$	1.09	0.82	0.43	0.41	$\Gamma(\Omega_{bb} \rightarrow \Omega'_{bc} l \bar{\nu}_l)$	1.16	0.83	0.48	0.51
$\Gamma(\Xi_{bb} \rightarrow \Xi_{bc} l \bar{\nu}_l)$	2.00	1.63	0.80	0.69	$\Gamma(\Omega_{bb} \rightarrow \Omega_{bc} l \bar{\nu}_l)$	2.15	1.70	0.86	0.92
$\Gamma(\Xi'_{bc} \rightarrow \Xi_{cc} l \bar{\nu}_l)$	1.36	0.88	1.10	–	$\Gamma(\Omega'_{bc} \rightarrow \Omega_{cc} l \bar{\nu}_l)$	1.36	0.95	0.98	–
$\Gamma(\Xi_{bc} \rightarrow \Xi_{cc} l \bar{\nu}_l)$	2.57	2.30	2.10	1.38	$\Gamma(\Omega_{bc} \rightarrow \Omega_{cc} l \bar{\nu}_l)$	2.58	2.48	1.88	1.54
$\Gamma(\Xi'_{bc} \rightarrow \Xi_{cc}^* l \bar{\nu}_l)$	2.35	1.70	2.01	–	$\Gamma(\Omega'_{bc} \rightarrow \Omega_{cc}^* l \bar{\nu}_l)$	2.35	1.83	1.93	–
$\Gamma(\Xi_{bc} \rightarrow \Xi_{cc}^* l \bar{\nu}_l)$	0.75	0.72	0.64	0.52	$\Gamma(\Omega_{bc} \rightarrow \Omega_{cc}^* l \bar{\nu}_l)$	0.76	0.74	0.62	0.56

TABLE 4. Electromagnetic decay widths (in units of 10^{-8} GeV) for unmixed states.

	This work	[9]		This work	[9]
$\Xi_{bcu}^* \rightarrow \Xi'_{bcu} \gamma$	4.04	0.28 ± 0.01	$\Omega_{bcs}^* \rightarrow \Omega'_{bcs} \gamma$	3.69	0.16 ± 0.01
$\Xi_{bcd}^* \rightarrow \Xi'_{bcd} \gamma$	4.04	0.28 ± 0.01			
$\Xi_{bcu}^* \rightarrow \Xi_{bcu} \gamma$	105	49 ± 9	$\Omega_{bcs}^* \rightarrow \Omega_{bcs} \gamma$	20.9	0.12 ± 0.02
$\Xi_{bcd}^* \rightarrow \Xi_{bcd} \gamma$	50.5	24 ± 4			
$\Xi'_{bcu} \rightarrow \Xi_{bcu} \gamma$	0.992	1.56 ± 0.08	$\Omega'_{bcs} \rightarrow \Omega_{bcs} \gamma$	0.568	1.26 ± 5
$\Xi'_{bcd} \rightarrow \Xi_{bcd} \gamma$	0.992	1.56 ± 0.08			

decay widths.

The results for $b \rightarrow c$ semileptonic decays in the unmixed case are shown in Table 3, where for comparison we also show the results obtained in Refs.[7, 8], within different relativistic approaches, and in the nonrelativistic calculation of Ref.[2]. Our results are in a global fair agreement with the ones in Ref.[7]. As for the other relativistic calculation in Ref.[8], the agreement is fair for transitions with a bc baryon in the initial state but there is an approximate factor of 2 discrepancy for transitions with a bc baryon in the final state. The nonrelativistic calculation in Ref.[2] also gives results that are roughly a factor of 2 smaller than ours for all decays. A very interesting feature of the decay widths shown in Table 3 is that they are very different for transitions involving Ξ_{bc} or Ξ'_{bc} (Ω_{bc} or Ω'_{bc}). This means, as suggested in Ref.[1], that mixing in those states, provided the admixture coefficients are large, can have a great impact on the decay widths.

In Table 4 we show our results for electromagnetic decays in the unmixed case. Branz et al. [9] have also studied electromagnetic decays of doubly heavy baryons within a relativistic constituent quark model. The agreement with our results is very poor in this case in part due to the different masses used in both calculations. The agreement improves in most cases if we divide out the $(M_I - M_F)^3$ mass factor discussed above. Still the differences are in the range 50-80%. This can be seen in Table 5.

$b \rightarrow c$ semileptonic decay widths involving the mixed states $\Xi_{bc}^{(1)}$, $\Xi_{bc}^{(2)}$ and $\Omega_{bc}^{(1)}$, $\Omega_{bc}^{(2)}$ are now given in Table 6. We see rather big changes from the values in Table 3 where unmixed states were used. Special attention deserves the $B_{bc}^{(2)} \rightarrow B_{cc}^*$ transitions where the width reduces by a large factor of 44 (54) for the $\Xi_{bc}^{(2)} \rightarrow \Xi_{cc}^*$ ($\Omega_{bc}^{(2)} \rightarrow \Omega_{cc}^*$) decay compared to the unmixed case. This can be easily understood by taking into account

TABLE 5. Electromagnetic decay widths, divided by $(M_I - M_F)^3$, (in units of $(10^{-5} \text{ GeV}^{-2})$ for unmixed states.

	This work	Branz <i>et al.</i>		This work	Branz <i>et al.</i>
$\Xi_{bcu}^* \rightarrow \Xi'_{bcu} \gamma$	73.6	57.0	$\Omega_{bcs}^* \rightarrow \Omega'_{bcs} \gamma$	72.8	58.3
$\Xi_{bcd}^* \rightarrow \Xi'_{bcd} \gamma$	73.6	57.0			
$\Xi_{bcu}^* \rightarrow \Xi_{bcu} \gamma$	333.9	471	$\Omega_{bcs}^* \rightarrow \Omega_{bcs} \gamma$	87.7	160
$\Xi_{bcd}^* \rightarrow \Xi_{bcd} \gamma$	160.6	231			
$\Xi'_{bcu} \rightarrow \Xi_{bcu} \gamma$	36.7	57.8	$\Omega'_{bcs} \rightarrow \Omega_{bcs} \gamma$	36.3	57.4
$\Xi'_{bcd} \rightarrow \Xi_{bcd} \gamma$	36.7	57.8			

TABLE 6. Semileptonic decay widths (in units of 10^{-14} GeV) for mixed states.

	This work	[2]		This work	[2]
$\Gamma(\Xi_{bb}^* \rightarrow \Xi_{bc}^{(1)} l \bar{\nu}_l)$	0.47	–	$\Gamma(\Omega_{bb}^* \rightarrow \Omega_{bc}^{(1)} l \bar{\nu}_l)$	0.48	–
$\Gamma(\Xi_{bb}^* \rightarrow \Xi_{bc}^{(2)} l \bar{\nu}_l)$	0.99	–	$\Gamma(\Omega_{bb}^* \rightarrow \Omega_{bc}^{(2)} l \bar{\nu}_l)$	1.06	–
$\Gamma(\Xi_{bb} \rightarrow \Xi_{bc}^{(1)} l \bar{\nu}_l)$	2.21	0.95	$\Gamma(\Omega_{bb} \rightarrow \Omega_{bc}^{(1)} l \bar{\nu}_l)$	2.36	0.99
$\Gamma(\Xi_{bb} \rightarrow \Xi_{bc}^{(2)} l \bar{\nu}_l)$	0.85	0.33	$\Gamma(\Omega_{bb} \rightarrow \Omega_{bc}^{(2)} l \bar{\nu}_l)$	0.91	0.30
$\Gamma(\Xi_{bc}^{(1)} \rightarrow \Xi_{cc} l \bar{\nu}_l)$	0.38	–	$\Gamma(\Omega_{bc}^{(1)} \rightarrow \Omega_{cc} l \bar{\nu}_l)$	0.37	–
$\Gamma(\Xi_{bc}^{(2)} \rightarrow \Xi_{cc} l \bar{\nu}_l)$	3.50	1.92	$\Gamma(\Omega_{bc}^{(2)} \rightarrow \Omega_{cc} l \bar{\nu}_l)$	3.52	1.99
$\Gamma(\Xi_{bc}^{(1)} \rightarrow \Xi_{cc}^* l \bar{\nu}_l)$	3.14	–	$\Gamma(\Omega_{bc}^{(1)} \rightarrow \Omega_{cc}^* l \bar{\nu}_l)$	3.14	–
$\Gamma(\Xi_{bc}^{(2)} \rightarrow \Xi_{cc}^* l \bar{\nu}_l)$	0.017	0.026	$\Gamma(\Omega_{bc}^{(2)} \rightarrow \Omega_{cc}^* l \bar{\nu}_l)$	0.014	0.013

that $B_{bc}^{(2)} \approx (|qc; 0\rangle \otimes |b; \frac{1}{2}\rangle)^{J=1/2}$. In the latter state the light and c quarks are coupled to spin 0, whereas in the B_{cc}^* the light and any of the c quarks are in a relative spin 1 state. In any spectator calculation, as the ones here and in Ref.[2], the amplitude for the $(|qc; 0\rangle \otimes |b; \frac{1}{2}\rangle)^{J=1/2} \rightarrow B_{cc}^*$ transition cancels due to the orthogonality of the different spin states of the spectator quarks in the initial and final baryons. The fact that $B_{bc}^{(2)}$ slightly deviates from $(|qc; 0\rangle \otimes |b; \frac{1}{2}\rangle)^{J=1/2}$ produces a non zero, but small, decay width.

Our results for the electromagnetic decay width for mixed states are enclosed in

TABLE 7. Electromagnetic decay widths (in units of 10^{-8} GeV) for mixed states.

	This work	[9]		This work	[9]
$\Xi_{bcu}^* \rightarrow \Xi_{bcu}^{(1)} \gamma$	6.05	0.15 ± 0.02	$\Omega_{bcs}^* \rightarrow \Omega_{bcs}^{(1)} \gamma$	0.31	$(1 \pm 1) \cdot 10^{-4}$
$\Xi_{bcd}^* \rightarrow \Xi_{bcd}^{(1)} \gamma$	0.12	$(2 \pm 2) \cdot 10^{-4}$			
$\Xi_{bcu}^* \rightarrow \Xi_{bcu}^{(2)} \gamma$	73.9	46 ± 10	$\Omega_{bcs}^* \rightarrow \Omega_{bcs}^{(2)} \gamma$	50.2	29 ± 3
$\Xi_{bcd}^* \rightarrow \Xi_{bcd}^{(2)} \gamma$	103	51 ± 6			
$\Xi_{bcu}^{(1)} \rightarrow \Xi_{bcu}^{(2)} \gamma$	12.4	14 ± 3	$\Omega_{bcs}^{(1)} \rightarrow \Omega_{bcs}^{(2)} \gamma$	8.52	21 ± 2
$\Xi_{bcd}^{(1)} \rightarrow \Xi_{bcd}^{(2)} \gamma$	20.9	31 ± 4			

TABLE 8. Electromagnetic decay widths, divided by $(M_I - M_F)^3$ in units of $(10^{-5} \text{ GeV}^{-2})$ for mixed states.

	This work	Branz <i>et al.</i>		This work	Branz <i>et al.</i>
$\Xi_{bcu}^* \rightarrow \Xi_{bcu}^{(1)} \gamma$	248	293	$\Omega_{bcs}^* \rightarrow \Omega_{bcs}^{(1)} \gamma$	12.7	0.8
$\Xi_{bcd}^* \rightarrow \Xi_{bcd}^{(1)} \gamma$	4.9	0.39			
$\Xi_{bcu}^* \rightarrow \Xi_{bcu}^{(2)} \gamma$	161.8	261	$\Omega_{bcs}^* \rightarrow \Omega_{bcs}^{(2)} \gamma$	146	218
$\Xi_{bcd}^* \rightarrow \Xi_{bcd}^{(2)} \gamma$	226	290			
$\Xi_{bcu}^{(1)} \rightarrow \Xi_{bcu}^{(2)} \gamma$	112	126	$\Omega_{bcs}^{(1)} \rightarrow \Omega_{bcs}^{(2)} \gamma$	124	215
$\Xi_{bcd}^{(1)} \rightarrow \Xi_{bcd}^{(2)} \gamma$	189	280			

Table 7. To the best of our knowledge ours was the first calculation which took into account the effect of the mixing in the electromagnetic decay width of bc -baryons. As for the unmixed case, we find a better qualitative agreement with the results of Ref. [9] when we show the decay widths divided by the $(M_I - M_F)^3$ mass factor, see Table 8.

CONCLUSIONS

We qualitatively confirm the findings in Refs. [1, 2] as to the relevance of hyperfine mixing in $b \rightarrow c$ semileptonic decays of doubly heavy baryons. Actual results differ by a factor of two. We find mixing is also very important for electromagnetic decays. In this latter case decay widths are very sensitive to the actual baryon masses. We have compared our results for the electromagnetic decay widths with those from Ref. [9]. Predictions are rather different, although a better agreement is found if the dependence of the width on $(M_I - M_F)^3$ is divided out. Electromagnetic decay studies demand an accurate determination of the masses.

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